Grade 10
Linear Equations in Two Variables

Choose correct answer(s) from the given choices

(1) Find the value of p and q if the following pair of equations have infinitely many solutions.
-6x - 60y -42 = 0
(p + 3q)x + (3p + 2q)y -14 = 0

a. p = -7, q = 3 **b.** p = -8, q = 2
c. p = -7, q = 2 **d.** p = -9, q = 2

(2) What will be the nature of lines representing a pair of linear equations that are inconsistent?

a. intersecting **b.** intersecting or coincident
c. parallel **d.** coincident

(3) On selling a holder-box at 40% gains and bowl-box at 50% gain, a seller gains $\$16$. If he sells the holder-box at 40% gain and bowl-box at 70% gain, he gains $\$22$. Find the actual price of holder-box and bowl-box .

a. $\$7.5, \$35$ **b.** $\$12.5, \$40$
c. $\$17.5, \$45$ **d.** $\$2.5, \$30$

(4) Solve the following pair of linear equations by the method of cross-multiplication.

$$px - qy = p + q$$
$$qx + py = q - p$$

a. $x = 0 \text{ and } y = -1$ **b.** $x = 1 \text{ and } y = -1$
c. $x = 0 \text{ and } y = 0$ **d.** None of these

(5) Solve the following system of linear equations by the method of cross-multiplication.

$$\frac{x}{m} - \frac{y}{n} = m + n$$
$$\frac{x}{m^2} - \frac{y}{n^2} = 7$$

a. $x = \dfrac{6m^2n - m^3}{-m + n}$ and $y = \dfrac{6mn^2 - n^3}{-m + n}$

b. $x = \dfrac{6m^2n - m^4}{-m - n}$ and $y = \dfrac{6mn^2 + n^3}{-m + n}$

c. $x = \dfrac{6n^3 - m^3}{-m^2 + n^2}$ and $y = \dfrac{6mn^2 - n^3}{-m + n}$

d. $x = \dfrac{6m^3n - m^4}{-m^2 + n}$ and $y = \dfrac{6m^2n^2 + n^3}{-m + n}$

Fill in the blanks

(6) Points R and S are $39\ km$ apart on a highway. A car starts from R and another car starts from S at the same time. If they travel in the same direction, they meet in $3\ hours$, but if they travel towards each other they meet in one hour. The speed of R is ______ km/hr and the speed of S is ______ km/hr.

Check True/False

(7) Equations **2x - 3y 0 = 0** and **-4x + 6y + k = 0** will have an unique solution for all real values of **k**.

○ True ○ False

Answer the questions

(8) The sum of two number is 41 and their difference is 9. Find the numbers.

(9) Write the condition for the given system of equations to be inconsistent.
$a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$

(10) It is given that the sum of digits of a two digit number is 12. If 18 is added to the number, the digits interchange their place. Find the number.

Solutions

(1) **b.** p = -8, q = 2

Step 1

From the equations, -6x - 60y -42 = 0 and (p + 3q)x + (3p + 2q)y -14 = 0, we notice that,
$a_1 = -6$, $b_1 = -60$, $c_1 = -42$
and $a_2 = p + 3q$, $b_2 = 3p + 2q$, $c_2 = -14$

Step 2

It is given that, the equations -6x - 60y -42 = 0 and (p + 3q)x + (3p + 2q)y -14 = 0 have infinitely many solutions.

Therefore, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{-6}{p + 3q} = \frac{-60}{3p + 2q} = \frac{-42}{-14}$

Step 3

On comparing first two terms,

$\Rightarrow \frac{-6}{p + 3q} = \frac{-60}{3p + 2q}$

We get, -6(3p + 2q) = -60(p + 3q)
⇒ 42p = -168q
⇒ p = -4 q

Step 4

Now, lets compare first and last term,

$\frac{-6}{p + 3q} = \frac{-42}{-14}$

We get, -6(-14) = -42(p + 3q)
⇒ 84 = -42p -126q
On substituting p = -4 q,
⇒ 84 = -42(-4 q) -126q
⇒ q = 2

Step 5

p = -4 q
⇒ -4 × 2
⇒ -8

Step 6

Hence, the value of p = -8
and q = 2.

(2) **c.** parallel

Step 1

When we say that a pair of lines form an inconsistent system of equations, it means that the pair of lines has no solution.

Step 2

Hence the lines are **parallel** to each other.

(3) **d.** $\$2.5$, $\$30$

Step 1

Let the cost price of the holder-box be $\$x$ and the cost of bowl-box be $\$y$ respectively.

Step 2

Now, let us find out the linear equation when holder-box is sold at 40 % and bowl-box at 50 % gain.

$$\begin{aligned} \text{Gain on holder-box } &= \$\frac{40x}{100} \\ \text{Gain on bowl-box } &= \$\frac{50y}{100} \\ \text{Net gain } &= \$\frac{40x}{100} + \$\frac{50y}{100} \\ \implies \frac{40x}{100} + \frac{50y}{100} &= 16 \\ \implies 40x + 50y = 1600 & \qquad \ldots(i) \end{aligned}$$

Step 3

Now, let us find out the linear equation when holder-set is sold at 40 % gain and bowl-set at 70 % gain.

$$\begin{aligned} \text{Gain on holder-set } &= \$\frac{40x}{100} \\ \text{Gain on bowl-set } &= \$\frac{70y}{100} \\ \text{Net gain } &= \$\frac{40x}{100} + \$\frac{70y}{100} \\ \implies \frac{40x}{100} + \frac{70y}{100} &= 22 \\ \implies 40x + 70y = 2200 & \qquad \ldots(ii) \end{aligned}$$

Step 4

Subtracting $eq(\text{i})$ from $eq(\text{ii})$, we get

$$20y = 600 \implies y = \frac{600}{20} = 30$$

Step 5

Substituting the value of $y = 30$ in $eq(\text{i})$, we get

$$\begin{aligned} & 40x + 50(30) = 1600 \\ \implies & 40x = 1600 - 1500 = 100 \\ \implies & x = \frac{100}{40} = 2.5 \end{aligned}$$

Step 6

Thus, the actual price of holder-box is $\$2.5$ and bowl-box is $\$30$.

(4) **b.** $x = 1$ and $y = -1$

Step 1

The given pair of equations is

$$px - qy = p + q \implies px - qy - (p+q) = 0$$
$$qx + py = q - p \implies qx + py - (q-p) = 0$$

Step 2

By cross-multiplication, we get

$$\frac{x}{\begin{matrix} -q & \nwarrow\!\!\!\!\searrow & -(p+q) \\ p & & -(q-p) \end{matrix}} = \frac{-y}{\begin{matrix} p & \nwarrow\!\!\!\!\searrow & -(p+q) \\ q & & -(q-p) \end{matrix}} = \frac{1}{\begin{matrix} p & \nwarrow\!\!\!\!\searrow & -q \\ q & & p \end{matrix}}$$

$$\implies \frac{x}{-q\times-(q-p) - p\times-(p+q)} = \frac{-y}{-(q-p)\times p + (p+q)\times q} = \frac{1}{p\times p - q\times(-q)}$$

$$\implies \frac{x}{q(q-p) + p(p+q)} = \frac{-y}{-p(q-p) + q(p+q)} = \frac{1}{(p^2+q^2)}$$

$$\implies \frac{x}{q^2 - qp + p^2 + pq} = \frac{-y}{-pq + p^2 + qp + q^2} = \frac{1}{(p^2+q^2)}$$

$$\implies \frac{x}{(q^2+p^2)} = \frac{-y}{(q^2+p^2)} = \frac{1}{(p^2+q^2)}$$

$$\implies \frac{x}{(q^2+p^2)} = \frac{1}{(p^2+q^2)} \text{ and } \frac{-y}{(q^2+p^2)} = \frac{1}{(p^2+q^2)}$$

$$\implies x = \frac{(q^2+p^2)}{(p^2+q^2)} \text{ and } y = -\frac{(q^2+p^2)}{(p^2+q^2)}$$

$$\implies x = 1 \text{ and } y = -1$$

Step 3

Hence, the required solution is $x = 1$ and $y = -1$.

(5) **a.** $x = \dfrac{6m^2n - m^3}{-m+n}$ and $y = \dfrac{6mn^2 - n^3}{-m+n}$

Step 1

The given system of equation is

$$\frac{x}{m} - \frac{y}{n} - (m+n) = 0 \qquad \ldots(i)$$
$$\frac{x}{m^2} - \frac{y}{n^2} - 7 = 0 \qquad \ldots(ii)$$

Step 2

Multiplying $eq(i)$ by mn, we get

$$nx - my - mn(m+n) = 0 \qquad \ldots(iii)$$

Similarly, multiplying $eq(ii)$ by m^2n^2, we get

$$n^2x - m^2y - 7m^2n^2 = 0 \qquad \ldots(iv)$$

Step 3

By cross multiplication method, we have

$$\frac{x}{\begin{matrix} -m & \times & -mn(m+n) \\ -m^2 & & -7m^2n^2 \end{matrix}} = \frac{-y}{\begin{matrix} n & \times & -mn(m+n) \\ n^2 & & -7m^2n^2 \end{matrix}} = \frac{1}{\begin{matrix} n & \times & -m \\ n^2 & & -m^2 \end{matrix}}$$

$$\Longrightarrow \frac{x}{7m^3n^2 - m^3n(m+n)} = \frac{-y}{-7m^2n^3 + mn^3(m+n)} = \frac{1}{-m^2n + mn^2}$$

$$\Longrightarrow \frac{x}{7m^3n^2 - m^4n - m^3n^2} = \frac{-y}{-7m^2n^3 + m^2n^3 + mn^4} = \frac{1}{-m^2n + mn^2}$$

$$\Longrightarrow \frac{x}{6m^3n^2 - m^4n} = \frac{-y}{-6m^2n^3 + mn^4} = \frac{1}{-m^2n + mn^2}$$

$$\Longrightarrow \frac{x}{6m^3n^2 - m^4n} = \frac{y}{6m^2n^3 - mn^4} = \frac{1}{-m^2n + mn^2}$$

$$\Longrightarrow \frac{x}{6m^3n^2 - m^4n} = \frac{1}{-m^2n + mn^2} \text{ and } \frac{y}{6m^2n^3 - mn^4} = \frac{1}{-m^2n + mn^2}$$

$$\Longrightarrow x = \frac{6m^3n^2 - m^4n}{-m^2n + mn^2} \text{ and } y = \frac{6m^2n^3 - mn^4}{-m^2n + mn^2}$$

$$\Longrightarrow x = \frac{6m^2n - m^3}{-m+n} \text{ and } y = \frac{6mn^2 - n^3}{-m+n}$$

Step 4

Hence, the value of x is $\dfrac{6m^2n - m^3}{-m+n}$ and the value of y is $\dfrac{6mn^2 - n^3}{-m+n}$.

(6) 26 13

Step 1

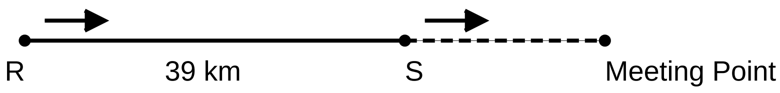
Let the speed of car starting from R be $x\ km/hr$ and starting from S be $y\ km/hr$.

We know that

$$Distance = Speed \times Time$$

Step 2

If the cars travel in the same direction, they meet in $3\ hours$.



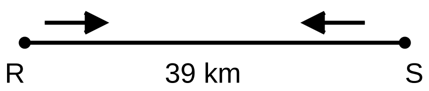
$\therefore$ Distance covered by car starting from point R in $3\ hours$ = $3x\ km$

and distance covered by car starting from point S in $3\ km$ = $3y\ km$

$$\therefore\ 3x - 3y = 39$$
$$\implies x - y = 13 \qquad \ldots(\text{i})$$

Step 3

If the cars travel in the opposite direction, they meet in $1\ hour$.



$\therefore$ Distance covered by car starting from point R in $1\ hour$ = $x\ km$

and distance covered by car starting from point S in $1\ hour$ = $y\ km$

$$\therefore\ x + y = 39 \qquad \ldots(\text{ii})$$

Step 4

Adding eq (i) and (ii), we have

$$2x = 52$$
$$\implies x = 26$$

Substituting $x = 26$ in eq (i), we get

$$26 - y = 13$$
$$\implies 26 - 13 = y$$
$$\implies 13 = y$$

Step 5

Thus, the speed of car starting from R is $26\ km/hr$ and starting from S is $13\ km/hr$.

(7) False

Step 1

On comparing equations with standard form of equations $\mathbf{a_1x + b_1y + c_1 = 0}$ and $\mathbf{a_2x + b_2y + c_2 = 0}$, we get,

$a_1 = -4$, $b_1 = 6$, $c_1 = k$,

and $a_2 = 2$, $b_2 = -3$, $c_2 = 0$

Step 2

Now, $\frac{a_1}{a_2} = \frac{-4}{2}$

and $\frac{b_1}{b_2} = \frac{6}{-3}$

Step 3

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the two lines represented by the equations **2x - 3y 0 = 0** and **-4x + 6y + k = 0** are parallel, and these equations will not have an unique solution, for all values of **k**.

Therefore, the answer is **False**.

(8) **25** and **16**

Step 1

Let the two numbers be x and y.

As the sum of two numbers is 41, we have

$$x + y = 41 \qquad \ldots (\text{i})$$

Also, the difference of two numbers is 9. So,

$$x - y = 9 \qquad \ldots (\text{ii})$$

Step 2

Adding equation (i) and (ii), we have

$$2x = 50$$
$$\implies x = 25$$

Substituting the value of x in equation (i), we have

$$25 + y = 41$$
$$\implies y = 41 - 25$$
$$\implies y = 16$$

Step 3

Thus, the two numbers are **25** and **16**.

(9) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Step 1

A pair of equations is said to be inconsistent if no solution exists that can satisfy both the equations.

Step 2

A pair of equations would have no solution if the ratio of coefficients of variables is same but is different from the ratio of the constants, i.e.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} ,$$

where a_1, b_1 and a_2, b_2 are the coefficients of the variables and c_1 and c_2 are the constants of the equations respectively.

Step 3

Hence, the condition for the given pair of equations to be inconsistent is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

(10) 57

Step 1

Let the ten's digit of the number be x and unit's digit be y. The number becomes 10x + y.
Now, it is given that the sum of digits of a two digit number is 12.
i.e. x + y = 12 ________________________(1)
It is also given that if 18 is added to the number, the digits interchange their place.
The number formed after interchanging the digits = 10y + x
So, the equation becomes (10x + y) + 18 = 10y + x
9x - 9y = -18
x - y = -2 ________________________(2)

Step 2

On adding the two equations,
2x + y - y = 12 - 2
2x = 10
x = 5

Step 3

Substituting the value of x in equation (1), we have:
x + y = 12
5 + y = 12
y = 12 - 5 = 7
Therefore, the number is **57**.